Demographic inference based on Site frequency spectrum (SFS) using fastsimcoal2

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Demographic history of populations

Past demographic events:

- Population split
- Migration events
- Changes in effective population sizes (expansions or bottlenecks)
- Temporal changes in migration rates and effective sizes

Population tree



Because of recombination, different regions of the genome can have different gene trees

Genome

- Demography is expected to affect the entire genome
- Natural selection acts on specific functional regions

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Gene trees in growing populations



Long external branches

- Coalescent rate is larger in smaller populations, and so we expect smaller intervals between coalescent events in smaller populations
- Coalescent rate is lower with a lower number of lineages, and so we expected larger • intervals between coalescent events as the number of lineages decrease

Stationary population gene trees at five genome regions



Figure 4.2 Five replicates of the coalescent process with constant population size for a sample of ten genes. Note the large variance in the time of the MRCA among replicates.





Figure 4.3 Five replicates of the coalescent with exponential growth, $\beta = 1000$, for a sample of n = 10 genes. Note the smaller variance in the time until the MRCA compared to the same quantity in Figure 4.2.

Hein et al (2004) Gene Genealogies, Variation and evolution

Gene trees for decreasing populations



- If we could observed directly the gene trees, we could easily reconstruct the population tree and the demographic history.
- But we do not observe gene trees...
- We can still learn about gene trees from the observed mutations and the allele frequencies in samples

Adding neutral mutations

The shape of neutral coalescent trees only depend on the population demography, and not on the mutational process. Assuming that all alleles have the same fitness, the mutational process can be modeled as an independent process superimposed on a realized coalescent tree.



Mutations just accumulate along the branches of the tree according to a **Poisson process** with rate $\lambda_i = \mu t_i$ for the *i*-th branch of length t_i . The Poisson process is stochastic but it should be immediately **obvious that long branches will carry more mutations than short branches**

Hein et al (2004) Gene Genealogies, Variation and evolution

We expect less diversity in a bottlenecked population

- Mutations accumulate along the branches.
- The longer a given branch the more likely it becomes that a mutation have happened on it.



Stationary Population

Bottleneck Population

same mutations

We expect less diversity in a bottlenecked population





- The SFS summarizes efficiently genome-wide data
- Assuming a single population 1Dimensional SFS

Outgroup ATACCG... Individual 1 ATACCG... Individual 2 ATTCGG... Individual 3 ATACGG...

> Observed SFS

Site frequency spectrum (SFS)

- The SFS summarizes efficiently genome-wide data
- Assuming a single population 1Dimensional SFS



Frequency of derived allele

- The SFS summarizes efficiently genome-wide data
- Assuming a single population 1Dimensional SFS



Frequency of derived allele

- The SFS summarizes efficiently genome-wide data
- Assuming a single population 1Dimensional SFS

The SFS ignores information about linkage. It is best suited for the study of many unlinked (or recombining) DNA sequences.

In a stationary population, the expected SFS relative frequencies are given by:

$$E(\xi_i) = rac{ heta}{i}$$
 Fu and Li, 1993



Frequency of derived allele

We can obtain the SFS from genotype call data

Genotypes:

- 0 homozygote for reference allele
- 1 heterozygote
- 2 homozygote for alternative allele

This can be done if we have enough depth of coverage (>10x)



Observed SFS is a vector (1 dimensional SFS):

derived allele frequency

Frequency	0	1	2	3	4	5	6	7	8	9	10
SNP count	0	2	1	1	0	0	0	0	0	0	0

SFS from genotype call data

Even if we have millions of SNPs we can summarize the genomic data to 10 numbers with the SFS!

The size of the SFS depends on the number of sampled individuals.



frequency derived allele

Observed SFS is a vector (1 dimensional SFS):

Frequency	0	1	2	3	4	5	6	7	8	9	10
SNP count	0	250,032	152,300	76,504	45,362	30,210	15,329	5,642	3,524	2,123	0

Coalescent and the SFS

- A recent population growth following a bottleneck leads to gene trees with long external branches
- Very few mutations in the internal branches
- Most mutations in long external branches are only found in one lineage, resulting in an excess of singletons



frequency derived alleles

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frequency derived alleles

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Exercise - SFS from gene trees at multiple loci

Consider the following gene trees that were simulated for 4 genes with 10,000 bp for a population that went through an expansion. Mutations are indicated with a star.



1. Summing across gene trees, what is the site frequency spectrum (SFS) compatible with the above gene trees?



2. Given the gene trees for the 4 genes, what can we learn about the time of expansion?

SFS depends on past demography





Natural selection also affects the SFS



Background selection (BGS) leads to patterns similar to population expansion.

Bank et al (2014) Trends in Genetics

Population structure

Migration events can be incorporated into gene trees.

Migration from Pop 2 to Pop 1, leads to lineages moving from Pop 1 to pop 2 backward in time.

At each generation, the probability of immigration into population 1 from population 2 is given by:

 $Pr(migrate) = n_1 * m$

Where n_1 is the number of lineages in population 1, and m is the immigration rate.



Site frequency spectrum from multiple populations (joint SFS)

- For a pair of populations 2D SFS
 - Count the SNPs have a frequency of the derived allele of *i* in population 1, and of *j* in population 2
- We can extend this to 3D SFS, 4D SFS, etc.



Derived vs Minor allele frequency spectrum

- So far, we have assumed that the allele frequency is the number of sequences with the derived allele frequency (unfolded SFS). We need information (outgroup) to determine the ancestral/derived state.
- If we do not have that information, we can work with the minor allele frequency (folded SFS). In this case, the allele with a lower frequency is treated as the reference.

Model based inference

- What is the model that best fits the data?
- What are the most likely parameters of each model?



Sousa and Hey (2013) Nat. Rev. Gen.

A model is represented by a population tree that reflects the past evolutionary history

Parameters:

- Population split times
- Migration rates
- Effective population sizes
- Temporal changes in migration rates and effective sizes
- Selective coefficient and type of selection (positive or negative)
- Mutation rate
- Recombination rate

Population tree



Demography

Genomic processes Selection

The SFS contains information about the demographic history of populations



Inferring the demographic history from the SFS

Genomic Data



- Effective sizes

Inferring the demographic history from the SFS

- The likelihood is easily computed based on the expected SFS under a given model
- There are different ways to obtain the expected SFS
 - Diffusion (forward in time)
 - Coalescent (backward in time)



- Effective sizes

Framework for demographic inference



Estimating the SFS from the coalescent

The probability of a SFS entry *i* can be estimated under a specific model θ from its expected coalescent tree as (Nielsen 2000) $E(t, | \theta)$

$$p_i = \frac{E(t_i \mid \theta)}{E(T \mid \theta)}$$

Where t_i is the total length of all branches directly leading to *i* terminal nodes, and *T* is the total tree length.

It gives the relative probability that if a mutation occurs on one of these b_i branches, it will be observed *i* times in the sample

This is true under the infinite sites model. No more than 1 mutation per site, back mutations not allowed!



Composite likelihood

Even though we can have linked sites, we assume that all sites are independent. Given S polymorphic sites (SNPs) out of L sites (Adams and Hudson, 2004) the composite likelihood is:

$$CL = \Pr(X \mid \theta) \propto \frac{P_0^{L-S} (1 - P_0)^S \prod_{i=1}^{n-1} \hat{p}_i^{m_i}}{\overline{\boldsymbol{n}}}$$

probability of no mutation on the tree probability of at least one mutation in the tree

These probabilities depend:

- Number of monomorphic sites
- A fixed and mutation rate



Composite likehood

Excoffier et al. (2013) PloS Genetics

Everything is relative



- The same expected SFS can be obtained in a large or small tree
- We need a mutation rate and the number of monomorphic sites to distinguish among the two!



Methods based on the SFS

Different ways to obtain the expected SFS p_i under different demographic models

- Coalescent-based
 - Multiple populations

 Fastsimcoal2 (Excoffier et al 2013 PLoS Genetics)
 Momi (Kamm et al 2015) and Momi 2
 Rarecoal (Schiffels et al 2016 Nat Genetics)
 - Single population Stairway plot (Liu and Fu, 2015 Nat Genetics)
- Diffusion-based

Dadi (Gutenkunst et al 2009 PLoS Genetics) Multipop (Lukic and Hey 2012 Genetics) Jouganous et al (2017) Genetics

fastsimcoal2 program

- Fastsimcoal2 can estimate parameters from the SFS using coalescent simulations
- Maximum (composite) likelihood method
- Uses a conditional expectation (CEM) maximization algorithm to find parameter combinations that maximize the likelihood
- It approximate the expected SFS by performing coalescent simulations (>50,000)

Estimating the SFS and likelihoods with coalescent simulations

This probability p_i can then be estimated on the basis of Z simulations as

 $\hat{p}_{i} = \frac{\sum_{j=k \in \Phi_{i}}^{Z} b_{kj}}{\sum_{j=1}^{Z} T_{j}}$ where b_{kj} is the length of the *k*-th compatible branch in simulation *j*.

These probabilities can then be used to compute the composite likelihood of a given model as (Adams and Hudson, 2004)

$$CL = \Pr(X \mid \theta) \propto P_0^{L-S} (1 - P_0)^S \prod_{i=1}^{n-1} \hat{p}_i^{m_i}$$

where *X* is the SFS in a population sample of size n, *S* is the number of polymorphic sites, *L* is the length of the studied sequence,

and P_0 is the probability of no mutation on the tree

Approximating the expected SFS with coalescent simulations

Β

Increasing the number of simulations improves the approximation of the expected SFS



Log10(expected SFS)

T=0.1 - Expected vs. Fastsimcoal SFS

Excoffier et al. (2013) PloS Genetics; Chen (2012) TPB

Properties of composite likelihoods

This composite likelihood (CL) is not a proper likelihood due to the non-independence of allele frequencies at linked sites.

- CL is maximized for the same parameters as full likelihood
- Can be used for parameter estimation
- Confidence intervals cannot be estimated from likelihood profile, need to bootstrap
- CL surface might be more complex than likelihood surface, and thus more difficult to explore and get the global maximum
- CL ignores information on linkage disequilibrium (recombination) between sites



Effective size (Ne)

Comparisons of approaches

Simulation of 20 Mb data



 N_A

m₂₁

N₁ _____

8/10

Protocol for parameter estimation

1. Get the observed SFS:

- derived SFS (DAF or unfolded SFS), when the ancestral state is known;
- minor allele frequency SFS (MAF or folded SFS) when the ancestral state is unknown
- 2. Define the **demographic model**
- **3. Estimate the parameters** repeat 50-100 runs, and selecting the run with maximum likelihood
- Bootstrap to obtain confidence intervals for each parameter – bootstrap 10-100 datasets, by repeating a few runs for each dataset
 - For datasets with linked sites use block-bootstrap, diving the genome into blocks

Potential problems

- Maximization of the CL is not trivial (precision of the approximation and convergence problems)
- Need to repeat estimations to find maximum CL
- Needs genomic data (several Mb), difficult to have genespecific estimates
- Next-generation sequencing data must have high coverage (>10x) to correctly estimate SFS

Problems with estimation of demographic parameters from SFS

Can one learn history from the allelic spectrum?

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> Received 17 March 2007 Available online 30 January 2008

Theoretical Population Biology

www.elsevier.com/locate/tpb

A demographic history with the same spectrum as a constant size population



The depth of coverage varied considerably across individuals



- Applying the same threshold for all individuals can lead to biases
- Apply a filter on DP for each individual

Effect of DP filters on the SFS Simulation study



Comparing models with composite likelihoods

- Fastsimcoal2 likelihood is "correct" if all SNPs are independent
- We can then compare the model likelihoods using Akaike Information Criterion (AIC)



Effective size (Ne)

Composite likelihood provide unbiased maximum likelihood parameter estimates, but the likelihoods are inflated

A strategy to compare models

- 1. Divide the dataset into LD blocks.
- 2. Create a dataset with all SNPs (including linked SNPs)
- For each model, obtain the parameters that maximize the likelihood (this is ok even with linked sites!) and the corresponding expected SFS
- 4. Create a dataset with "independent" SNPs (1 SNP per RAD tag)
- Given the expected SFS of each model, compute the "correct" likelihood for each model with the dataset with independent SNPs
- 6. Compare models with AIC



Protocol for model comparison based on AIC when we have independent SNPs

- Get the observed SFS
- Define the alternative models
- Perform 50-100 runs under each model
- Select the runs with maximum likelihood under each model
- Compute the AIC (Akaike information critera) for each model
- Select the model with minimum AIC

FASTSIMCOAL2 INPUT FILES

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PGDH18

Examples of observed SFS

1PopExpIr	nst20Mb_DA	Fpop0.	obs							
1 observa	ations									
d0_0	d0_1	d0_2	d0_3	d0_4	d0_5	d0_6	d0_7	d0_8	d0_9	d0_10
19973842	24630	810	173	145	111	88	84	61	56	0

2PopDivM	igr20Mb_jo	intDAF	'pop1_0	.obs		
1 observ	1 observations					
	d0_0	d0_1	d0_2	d0_3	d0_4	d0_5
d1_0	19985747	8350	1628	360	62	8
d1_1	9660	0	0	0	0	
d1_2	4790	0	0	0	0	
d1 3	3280	0	0	0	0	
d1 4	2490	0	0	0	0	
d1 5	1760	13	18	13	19	
d1_5	1760	13	18	13	19	

2PopDiv2	0Mb_jointD	AFpop1	_0.obs			
1 observ	ations					
	d0_0	d0_1	d0_2	d0_3	d0_4	d0_5
d1_0	19985547	8211	1415	316	55	10
d1_1	1266	101	37	16	5	1
d1_2	61142	20	8	2	0	
d1 3	48631	12	5	0	0	
d1_4	47915	9	2	3	1	
d1_5	1189	46	22	19	18	0

Parameter estimation settings files



Additional files necessary to estimate parameters

Estimation file

11	opExpIn	st20Mb/1	opExp	Inst	20Mb.est					
// Search ranges and rules file // ***********************************										
[P	ARAMETERS]								
//	#isInt? #	name #dis	t.#min	#max						
//	all Ns ar	e in number	or hap	loid	individuals					
Ţ	NPOP	logunif	1000	le/	output					
1	NANC	logunif	10	1e5	output					
1	TEXP	unif	10	1e5	output					
[R	ULES]									
[C	OMPLEX PA	RAMETERS]								
0	RESIZE	= NANC/NPO	P :	hide						

Template file

1PopExpInst20Mb/1PopExpInst20Mb.tpl //Parameters for the coalescence simulation program : fsimcoal2.exe 1 samples to simulate : //Population effective sizes (number of genes) NPOP //Samples sizes and samples age 10 //Growth rates: negative growth implies population expansion 0 //Number of migration matrices : 0 implies no migration between demes 0 //historical event: time, source, sink, migrants, new deme size, new growth rate, migration matrix index 1 historical event TEXP 0 0 0 RESIZE 0 0 //Number of independent loci [chromosome] 1 0 //Per chromosome: Number of contiguous linkage Block: a block is a set of contiguous loci //per Block:data type, number of loci, per generation recombination and mutation rates and optional parameters FREO 1 0 2.5e-8 OUTEXP

INPUT files for fastsimcoal2: Defining an evolutionary model with TPL files



TPL files

TPL define the model using parameter tags.

These files are very important! Check carefully all the definitions. Errors in the TPL file are difficult to detect and imply the model specification is incorrect! This means that all inferences will be wrong, and also that all parameter estimates will be incorrect!

Defining population sizes and sample sizes



Population effective sizes are given in number of gene copies. For a diploid species with N=500 individuals, this corresponds to a 2N=1000 gene copies, as each individual carries two gene copies at any given site. The sample size is also given in gene copies. The value of 6

means that we sampled 3 diploid individuals.

Ind. 1 (2 gene copies)
Ind. 2 (2 gene copies)
Ind. 3 (2 gene copies)

TPL files

MIGRATION



The migration matrix can be asymmetric, and in the case the entry m_{ij} list the **migration rates backward in time** from population *i* to population *j*. The abovementioned matrix states that, for each generation backward in time, any gene from population 0 has probability MIG_01 to be sent to population 1, and that a gene from population 1 has a probability MIG_10 to move to population 0.

If no migration matrix is defined, no migration is assumed between populations.

1PopStationary10Loci.par

//Number of migration matrices : 0 implies no migration between demes 0 $\,$

A note on looking backward in time

Assuming that we look forward in time and that the size of the arrows are proportion to the migration rate, to what model does the following migration matrix corresponds to?

```
//Number of migration matrices : 0 implies no migration between demes
1
//migration matrix
0.000 0.005
0.001 0.000
```



A note on looking backward in time

Assuming that we look forward in time and that the size of the arrows are proportion to the migration rate, to what model does the following migration matrix corresponds to?



Historical events can be used to:

- Change the size of a given population
- Change the growth rate of a given population
- Change the migration matrix to be used between populations
- Move a fraction of the genes of a given population to another population. This amounts to implementing a (stochastic) admixture or introgression event.
- Move all genes from a population to another population. This amounts to fusing two populations into one looking backward in time.
- One or more of these events at the same time

Defining the historical events is crucial to have a correct model!

Historical events (backward in time) Each historical event is coded with a line with the following arguments

time, source, sink, migrants, new deme size, new growth rate, migration matrix index



500 generations ago, 100% (migrants=1.0) of lineages in pop0 (source =0) migrated to **pop1** (sink=1). The size of the sink (pop1) remained the same (new deme size=1.0, i.e. N2=2000). The new growth rate is zero. The migration rate that is active after the event is given in the migration matrix 1.

Historical events (backward in time) Each historical event is coded with a line with the following arguments

time, source, sink, migrants, new deme size, new growth rate, migration matrix index



Change the size of a given population



- 1000 generations ago, 0% (migrants=0) of lineages in pop0 (source) migrated to pop1 (sink). This means that 100% of lineages remained in pop0.
- The sink population (pop0) has a size 1000 larger after the event (new size=1000). Given that N0=500 diploids at time zero, it implies that NA=500000 diploids.
- The migration matrix valid after the event is the migration rate 0. Since it is not defined it imples no migration.





Change the migration matrix to be used between populations

2PopDivMigr10Loci.par
//Number of migration matrices : 0 implies no migration between demes
2
//Migration matrix 0
0 0
1e-4 0
//Migration matrix 1: No migration
0 0
0 0
//historical event: time, source, sink, migrants, new RELATIVE deme size, new growth rate, migration
matrix index
2 historical event
1000 0 0 1 0 1
5000 1 0 1 1.5 0 1

- At generation 1000 in the past, 0% (migrants=0) of lineages migrated from pop0 (source=0) to pop1 (sink=0).
- After the historical event, the deme size of the sink population (pop1) remained the same (new deme size=1).
- After the historical event the growth rate was set to zero.
- After the historical event the migration rate matrix was set to matrix 1, i.e. no migration between populations.



Population split (merge populations going backwards in time)



- At generation 5000 in the past, 100% (migrants=1) of lineages migrated from pop1 (source=1) to pop0 (sink=0).
- After the population split, the deme size of the sink population (pop0) is 1500 (new deme size=1500/20000=0.075).
- After the historical event the growth rate of the sink population pop0 is zero.
- After the historical event the migration rate matrix was set to matrix 1, i.e. no migration between populations.



Launching parameter estimations



Observed SFS file must have the same name as template file and extension _DAFpop0.obs. e.g. 1PopExpInst20Mb_DAFpop0.obs